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Color class domination and chromatic polynomial for ir-coloring and ND-coloring in fuzzy graphs

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Abstract

Let G be a fuzzy graph. A family $\Gamma^f = \{\gamma_1^f, \gamma_2^f, ..., \gamma_k^f\}$ of fuzzy sets on a set V is called k-fuzzy coloring of $V = (V, \sigma, \mu)$ if i) $\bigcup \Gamma^f = \sigma$ ii) $\gamma_i^f \cap \gamma_j^f = \phi$ iii) for every strong edge (x, y) (that is .,$\mu(xy) > 0$) of G, $\min\{\gamma_i^f(x), \gamma_i^f(y)\} = 0$ $(1 \le i \le k)$. The minimum number of k for which there exists a k-fuzzy coloring is called fuzzy chromatic number of G and is denoted by $\psi^f(G)$. Then Γ^f is the partition of independent sets of vertices of G in which each sets has the same color is called the fuzzy chromatic partition. A fuzzy dominator coloring of a fuzzy graph G is a proper fuzzy coloring of G in which every vertex of G dominates every vertex of at least one color class. The minimum number of colors required for a fuzzy dominator coloring of G is called the fuzzy dominator chromatic number (FDCN) and is denoted by $\psi_d^f(G)$. In this chapter , we introduce a new class of color partition and their related concepts. Also, we extensively studied the concept of chromatic polynomial for irregular fuzzy coloring and fuzzy neighborhood distinguished coloring.

Keywords and phrases: Fuzzy dominator coloring , fuzzy irregular dominator coloring , fuzzy neighborhood distinguished coloring, dom^f ND-chromatic, Chromatic polynomial for FNDC.

Subject Classification: 05C72

Introduction:

The concept of fuzzy sets and fuzzy relations was first introduced by L.A.Zadeh in 1965 [1]. Fuzzy graphs were introduced by Rosenfeld [2]. Rosenfeld has described the fuzzy analogue of several graph theoretic concept like paths,cycles, tree and connctedness and established some properties on them. Graph coloring is the most studied problem of combinatorial optimization. As a advancement fuzzy coloring of fuzzy graph was defined by authors Elashchi and Onagh in 2004, and later developed by them as fuzzy vertex coloring [3] in 2006. Harary and Plantholt [4] introduced yet another way to distinguished the vertices of a graph G by assigning colors to the edges of G in such a way thatfor every two verticesof G, one of the vertices is incident with an edge assigned one of these colors that the other vertex is not.

Graph coloring and domination are two major areas in graph theory that have been well studied. Hedetniemi et al [5,6] introduced the concept of dominator partition and dominator coloring of a graph. Ramar.R and V. Swaminathan [7] introduced the concept of neighborhood distinguished coloring in graphs. Here , we studied about fuzzy dominator coloring. Also ,in this paper we are introducing the concept of fuzzy irregular dominator coloring and chromatic polynomial for fuzzy neighborhood distinguished coloring and their related concepts.

Preliminaries

Definition :2.1

A fuzzy graph $G=(\sigma,\mu)$ is a pair of function $\sigma:V\rightarrow[0,1]\ \&\ \mu:VXV\rightarrow[0,1]$ where for al $u,v\in V$ we have $\mu(u,v)\le\sigma(u)\wedge\sigma(v)$

Definition: 2.2

The order p and size q of the fuzzy graph $G=(\sigma,\mu)$ are defined to be $p=\sum_{x\in V}\sigma(x)\ \&\ q=\sum_{xy\in E}\mu(xy)$

Definition: 2.3

The subset D of V is said to be fuzzy dominating set if every vertex $u\in V(G)$ there exist a vertex $v\in V-D$ such that $uv\in E(G)$ and $\mu(uv)\le\sigma(u)\wedge\sigma(v)$. The minimum cardinality of fuzzy dominating set is denoted by $\gamma^f(G)$.

Definition: 2.4

Let G be a fuzzy graph. A subset S of G is said to be fuzzy independent set of G if there exists no $uv\in S$ such that $\mu(uv)\le\sigma(u)\wedge\sigma(v)$. The maximum cardinality of such fuzzy independent set is called fuzzy independence number and is denoted by β_0^f

Definition: 2.5

The union of two fuzzy graphs G_1 and G_2 is defined as a fuzzy graph $G=G_1\cup G_2:(\sigma_1\cup\sigma_2,\mu_1\cup\mu_2)$ defined by

$$(\sigma_1\cup\sigma_2)(u)=\begin{cases}\sigma_1(u), if u\in V_1-V_2 and\\ \sigma_2(u), if u\in V_2-V_1 and\end{cases}$$

$$(\mu_1\cup\mu_2)(u)=\begin{cases}\mu_1(uv), if uv\in E_1-E_2 and\\ \mu_2(uv), if uv\in E_2-E_1 and\end{cases}$$

Definition: 2.6

Let G be a fuzzy graph. A family $\Gamma^f=\{\gamma_1^f,\gamma_2^f,\ldots,\gamma_k^f\}$ of fuzzy sets on a set V is called k-fuzzy coloring of $V=(V,\sigma,\mu)$ if i) $\cup\Gamma^f=\sigma$ ii) $\gamma_i^f\cap\gamma_j^f=\phi$ iii) for every strong edge (x,y) (that is .,$\mu(xy)>0$) of G, $\min\{\gamma_i^f(x),\gamma_i^f(y)\}=0$ $(1\le i\le k)$. The minimum number of k for which there exists a k-fuzzy coloring is called fuzzy chromatic number of G and is denoted by $\psi^f(G)$.

Definition: 2.7

Γ^f is the partition of independent sets of vertices of G in which each sets has the same color is called the fuzzy chromatic partition.

Main definition and results:

Definition: 3.1 Fuzzy dominator coloring

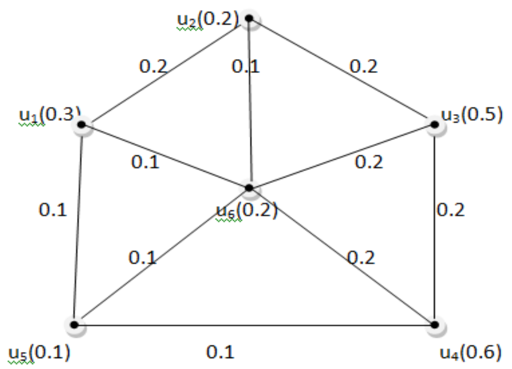
A fuzzy dominator coloring of a fuzzy graph G is a proper fuzzy coloring of G in which every vertex of G dominates every vertex of at least one color class . The minimum number of colors required for a fuzzy dominator coloring G is called the fuzzy dominator chromatic number (FDCN) and is denoted by $\psi_d^f(G)$

Example: 3.2

1. If G is fuzzy complete graph then $\psi_d^f(G)=n$
2. For complete fuzzy bipartite graph, $\psi_d^f(G)=2$
3. For path P_n:

$$\psi_d^f(P_n)=\begin{cases}1+\left\lceil\frac{n}{3}\right\rceil if n=2,3,4,5,7\\ 2+\left\lceil\frac{n}{3}\right\rceil otherwise\end{cases}$$

4. For star graph $K_{1,n},\psi_d^f(K_{1,n})=2$
5. For cycle $C_n,\psi_d^f(C_n)=\begin{cases}\left\lceil\frac{n}{3}\right\rceil if n=4\\ \left\lceil\frac{n}{3}\right\rceil+1 if n=5\\ \left\lceil\frac{n}{3}\right\rceil+2 otherwise\end{cases}$
6.



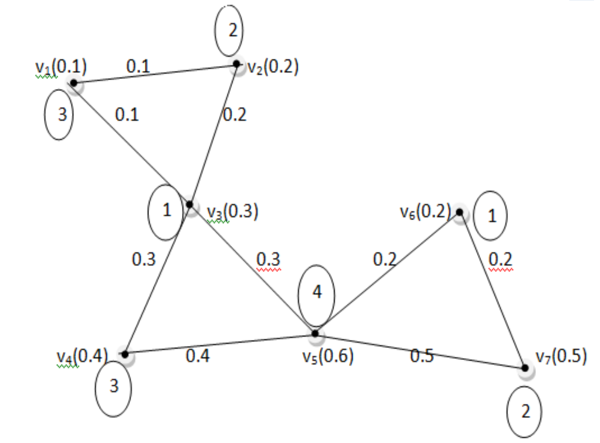
The color partition $\pi^f=\{\{v_1,v_4\},\{v_3\},\{v_6\},\{v_2,v_5\}\}$ and $\psi_d^f(G)=4$

Corollary: 3.3

For any fuzzy graph G, if $\gamma^f(G)=1$, then $\psi_d^f(G)=\psi^f(G)$

Definition: 3.4 Irregular coloring of fuzzy graph

A proper fuzzy coloring of a graph G is an irregular fuzzy coloring if whenever two vertices have same color, their neighborhood have different color pattern. The minimum cardinality of fuzzy ir-color partition is denoted by $\psi_{ir}^f(G)$

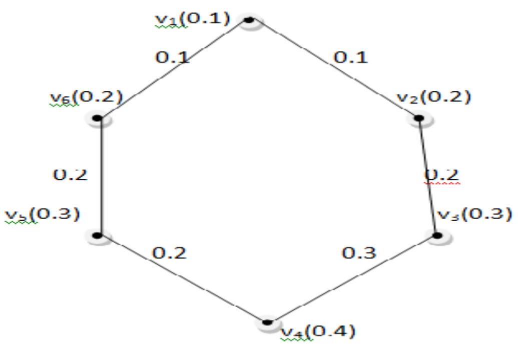


Example: 3.5

Definition: 3.6 Fuzzy ir-dominator coloring

Let G be a fuzzy graph. A proper fuzzy ir-color partition $\pi^f=\{V_1,V_2,...,V_k\}$ is called fuzzy dominator ir-color partition if every color class dominates a other color class. The mimimum cardinality of fuzzy dominator ir-color partition of G is called fuzzy ir-color dominator number of G and is denoted by $\psi_{ir}^{fd}(G)$.

Example: 3.7



A fuzzy ir-color dominator partition is $\pi^f=\{\{v_1,v_4\},\{v_3\},\{v_5\},\{v_2,v_6\}\}$ and $\psi_{ir}^{fd}(G)=4$

Remark: 3.8

- $\psi^f(G)\le\psi^{fd}(G)\le\psi_{ir}^{fd}(G)$
- $\psi^f(G)\le\psi_{ir}^f(G)\le\psi_{ir}^{fd}(G)$
- $\psi_{ir}^{fd}(G)\ge\max\{\psi_{ir}^f(G),\gamma^f(G)\}$
- $\psi_{ir}^{fd}(G)\le\psi_{ir}^f(G)+\gamma^f(G)$

Proof :

Let $\pi^f=\{V_1,V_2,...,V_k\}$be a ψ_{ir}^f-partition of G. Let $D=\{v_1,v_2,...,v_\gamma\}$ be a minimum fuzzy dominating set such that $\mu(v_iv_{i+1})\le\sigma(v_i)\wedge\sigma(v_{i+1})$, for all $i=1,2,...,\gamma$. Let $\pi_1^f=\{V_1',V_2',...,V_k',\{v_1\},\{v_2\},...,\{v_\gamma\}\}$ where the vertices $v_1,v_2,...,v_\gamma$ are removed from $V_1,V_2,...,V_k$ resulting in $V_1',V_2',...,V_k'$. Let $u\in V-D$. Since D is a dominating set of G , u is defined to some vertex of D , say v_i such that $\mu(uv_i)\le\sigma(u)\wedge\sigma(v_i)$. Then u dominates the color class $\{v_i\}$. Thus every vertex in V-D dominates a color class in π_1^f. Every vertex in D dominates itself and hence a color class in π_1^f. Thus π_1^f is a fuzzy dominator color partition of G. $C_{\pi_1}^f(v_i),1\le i\le\gamma$ will differ from $C_{\pi_1}^f(x)$ for any $x\in V-\{v\}$ in first place.

Suppose x and y belongs to V-D. If x and y belongs to different classes in π^f then $C_{\pi_1}^f(x)$ and $C_{\pi_1}^f(y)$ differ in the first place. If x and y belongs to the same class in π^f then there exists V_j such that $|N^f(x)\cap V_j|\ne|N^f(y)\cap V_j|$. If $|N^f(x)\cap V_j'|\ne|N^f(y)\cap V_j'|$ in π_1^f, there may exist some V_j such that x is adjacent with v_j and y is not adjacent with u_j and $u_j\in V_j$. Hence $C_{\pi_1}^f(x)$, $C_{\pi_1}^f(y)$ differ in the (k+j)th place . Hence π_1^f is an fuzzy ir-color partition . Since π_1^f us a fuzzy dominator color partition of G and also an fuzzy ir-partition of G, π_1^f is an fuzzy ir-color dominator partition of G. Therefore $\psi_{ir}^{fd}(G)\le|\pi_1^f|=|\pi^f|+\gamma^f(G)=\psi_{ir}^f(G)+\gamma^f(G)$.

Bounds for $\psi_{ir}^{fd}(G)$:

- $2\le\psi_{ir}^{fd}(G)\le n$

Let G be a connected fuzzy graph with vertices $n\ge 2$. Then $\psi_{ir}^{fd}(G)=2$ if and only if G=k_2

Theorem: 3.9

Let G be a connected fuzzy graph with vertices $n(\geq 4)$. Then $\psi_{ir}^{fd}(G)=n$ if and only if G=K_n (or) any two non-adjacent vertices in G have the same neighbor.

Proof

Let $n\geq 4$. Suppose $\psi_{ir}^{fd}(G)=n$.

To prove : Any two non-adjacent vertices have the same neighbor.

Suppose not. Assume $G\neq K_n$. Then there exist two non adjacent vertices say v_1,v_2.

Let us take the partition $\pi^f=\{\{v_1,v_2\},\{v_3\},....,\{v_n\}\}$. Clearly π^f is an fuzzy ir-dominator coloring of G. Thus $\psi_{ir}^{fd}(G)\leq|\pi^f|=n-1$,which is contradiction to the fact $\psi_{ir}^{fd}(G)=n$. Therefore either $G=K_n$ or any two non adjacent vertices have the same neighbor in G.

Conversely, suppose $G=K_n$ or any two non adjacent vertices have the same neighbor . Obviously $\psi_{ir}^{fd}(G)=n$.

Remark :3.10

- Let G be a connected fuzzy graph with vertices n=3 . Then $G=K_3 or P_3$ and $\psi_{ir}^{fd}(G)=3$.
- Let G be a connected fuzzy graph with vertices n=2. Then $G=K_2$ and $\psi_{ir}^{fd}(G)=2$.
- Let $G=K_n-e$. Then $\psi_{ir}^{fd}(G)=n$.
- Let G be a connected fuzzy graph with vertices $n\geq 2$. If every two non-adjacent vertices have the same neighbor then $\psi_{ir}^{fd}(G)=n$.
- If G is a complete multipartite fuzzy graph with n vertices , then $\psi_{ir}^{fd}(G)=n$.

Theorem: 3.11

Let G be a fuzzy graph . Let v_1,v_2 and v_3,v_4 be two pairs of non-adjacent vertices having different neighbor. Then $\psi_{ir}^{f}(G)\leq n-2$.

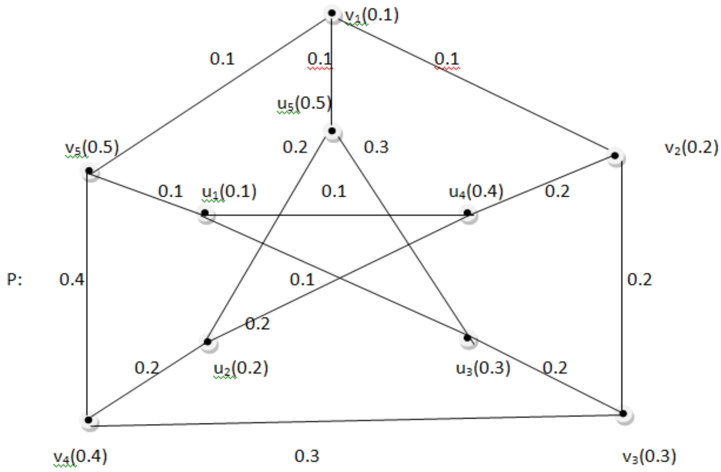
Proof

Let $\pi^f=\{\{v_1,v_2\},\{v_3,v_4\},\{v_5\}....,\{v_n\}\}$. Since v_1,v_2 and v_3,v_4 are two pairs of non adjacent vertices having different neighbors, then $\psi_{ir}^{fd}(G)\leq|\pi^f|=n-2$

Definition: 3.12 Dominated ir-color class partition in fuzzy graphs

Let G be a fuzzy graph. Let the fuzzy ir-color partition of V(G) be $\pi^f=\{V_1,V_2,...,V_k\}$. Then π^f is said to be dominated fuzzy ir-color partition of G if every fuzzy color class V_i is dominated by a vertex of V. Since $\pi^f=\{\{u_1\},\{u_2\},...,\{u_n\}\}$ is a fuzzy ir-color partition which is also a dominated fuzzy ir-color partition , the existence of such a dominated fuzzy ir-color partition is guaranteed. The minimum cardinality of dominated fuzzy ir-color partition is denoted by dom^f ir- color class partition number of G and is denoted by $\psi_{ir}^{fcd}(G)$

Example: 3.13



1. $\pi^f=\{\{v_1,u_2,v_3\},\{v_5,u_5,v_2\},\{v_4,u_1\},\{u_4\},\{u_3\}\}$ and $\psi_{ir}^{fcd}(P)=5$
2. $\psi_{ir}^{fcd}(K_n)=n$
3. $\psi_{ir}^{fcd}(K_{1,m})=m+1$

Remark: 3.14

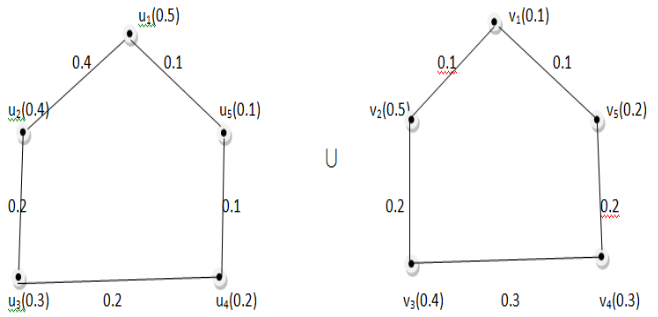
- Let G be a disconnected fuzzy graph with components $G_1,G_2,....,G_k$ then $\psi_{ir}^{f}(G)\leq\sum_{i=1}^{k}\psi_{ir}^{f}(G_i)$
- $\psi_{ir}^{fcd}(G)=\sum_{i=1}^{n}\psi_{ir}^{fcd}(G)$

Remark: 3.15

There are graphs G with components G_1 and G_2 in which $\psi_{ir}^{f}(G) < \psi_{ir}^{f}(G_1) + \psi_{ir}^{f}(G_2)$.

For , let $G = C_5 \cup C_5$

G:

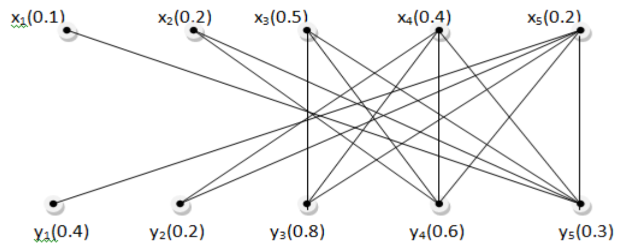


Let $\pi^f = \{\{u_1, u_4, v_1, v_3\}, \{u_2, u_5, v_2, v_5\}, \{u_3, v_4\}\}$. Then π^f is an ir-color partition of G. Thus $\psi_{it}^{f}(G) = 3 \le \psi_{ir}^{f}(G_1) + \psi_{ir}^{f}(G_2)$

Remark: 3.16

For each even integer n=2k (k≥2). Let F_n be the bipartite fuzzy graph with partite sets $X = \{x_1, x_2, ..., x_k\} \,\&\, Y = \{y_1, y_2, ..., y_k\}$

Let x_1 be joined with y_k, x_2 be joined with $y_k \,\&\, y_{k-1}, \ldots,$ x_k be joined with y_k, $y_{k-1}, \ldots, y_1$ such that $\mu(x_i y_i) \le \sigma(x_i) \wedge \sigma(y_i) \forall i = 1,2,...,k$. Consider the fuzzy graph F_{10}.



Here $\mu(x_i y_i) \le \sigma(x_i) \wedge \sigma(y_i) \forall i = 1,2,...,k$

Therefore $\psi_{ir}^{f}(F_{10}) = 2$. In general $\psi_{ir}^{f}(F_n) = 2$

Theorem: 3.17

$\psi_{ir}^{fd}(F_n) = 4$

Proof

Let F_n be the bipartite fuzzy graph [Remark:3.16]

Let $\pi^f = \{\{x_k\}, \{y_k\}, \{x_1, x_2 ...., x_{k-1}\}, \{y_1, y_2, ..., y_{k-1}\}\}$. Let π^f is an fuzzy ir-dominator color partition and hence $\psi_{ir}^{fd}(F_n) \le 4$. If $\psi_{ir}^{fd}(F_n) = 2$, then $\pi_1^f = \{\{x_1, x_2 ...., x_k\}, \{y_1, y_2, ..., y_k\}\}$ is an unique fuzzy ir-partition. But it is not an fuzzy ir-dominator partition. Since $x_1 \,\&\, y_1$ are adjacent with exactly one vertex and $N^f(x_1) \ne N^f(y_1)$, there exist at least two singleton elements in π_2 and they are $\{x_k\} \,\&\, \{y_k\}$ or $\{x_1\} \,\&\, \{y_1\}$. Also $|\pi_2| = 3$, the remaining vertices are in a single class which is not possible, since there is adjacency in the remaining vertices. Thus $\psi_{ir}^{fd}(F_n) \ge 4$ Therefore , $\psi_{ir}^{fd}(F_n) = 4$

Fuzzy neighborhood distinguished coloring graphs:

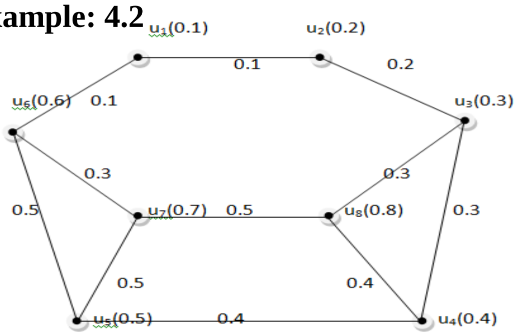
Definition: 4.1

Let $\pi^f = \{V_1, V_2, ..., V_k\}$ be a proper fuzzy coloring partition of a fuzzy graph. Fixing this order of π^f, for each $u \in V(G)$, we assign a code denoted by $C^f(u)$ or $C_\pi^f(u)$ as

$$C^f(u) = \min\{\{\sigma(N(u) \cap V_i)\}, i = 1,2,...,k / \mu(uv) \le \sigma(u) \wedge \sigma($$

. Then π^f is called fuzzy neighborhood distinguished coloring graph(FNDC).

Example: 4.2



ψ^{FNDC}-partition of G is $\pi^f = \{\{u_1, u_8, u_5\}, \{u_2, u_4, u_6\}, \{u_3, u_7\}\}$

Codes of the vertices with respect to π^f are :

$C_\pi(u_1) = (0,0.2,0)$, $C_\pi(u_2) = (0.1,0,0.3)$, $C_\pi(u_3) = (0.8,0.2,0)$, $C_\pi(u_4) = (0.5,0,0.3)$, $C_\pi(u_5) = (0,0.4,0.7)$, $C_\pi(u_6) = (0.1,0,0.7)$, $C_\pi(u_7) = (0.5,0.6,0)$, $C_\pi(u_8) = (0,0.4,0.7)$

Thus codes are distinct and π^f is FNDC- partition of G.

Theorem: 4.3

Let G be a fuzzy graph with components G_1 and G_2. Then $\psi_{ir}^{f}(G) \geq \max\{\psi_{ir}^{f}(G_1), \psi_{ir}^{f}(G_2)\}$

Proof

Let $\psi_{ir}^{f}(G) = t$. Let the fuzzy ir-color partition of G be $\pi^{f} = \{W_1, W_2, ..., W_t\}$. Let $X_k = W_k \cap V_1 \,\&\, Y_k = W_k \cap V_2, 1 \leq k \leq t$ where $V_1 = V(G_1) \,\&\, V_2 = V(G_2)$. Let $\pi_1^{f} = \{X_1, X_2, ..., X_t\}$ and $\pi_2^{f} = \{Y_1, Y_2, ..., Y_t\}$. Let $u, v \in V(G_1)$. Then $C_{\pi}^{f}(u) = (a_0, a_1, ..., a_t) \,\&\, C_{\pi}^{f}(v) = (b_0, b_1, ..., b_t)$. Since π^{f} is distinguishing, $C_{\pi}^{f}(u) \neq C_{\pi}^{f}(v)$ and $C_{\pi}^{f}(u) \neq C_{\pi_1}^{f}(u)$ for any $u \in V(G_1)$.

Therefore , $\psi_{ir}^{f}(G_1) \leq |\pi_1| = |\pi| = \psi_{ir}^{f}(G)$

Similarly, $\psi_{ir}^{f}(G_2) \leq \psi_{ir}^{f}(G)$

Therefore , $\psi_{ir}^{f}(G) \geq \max\{\psi_{ir}^{f}(G_1), \psi_{ir}^{f}(G_2)\}$

Example: 4.4

$$\psi_{ir}^{f}(P_4 \cup C_4) = 4 = \max\{\psi_{ir}^{f}(P_4), \psi_{ir}^{f}(C_4)\}$$

Remark: 4.5

i) $\psi^{f}(G) \leq \psi_{ir}^{f}(G) \leq \psi_{ir}^{fcd}(G)\}$

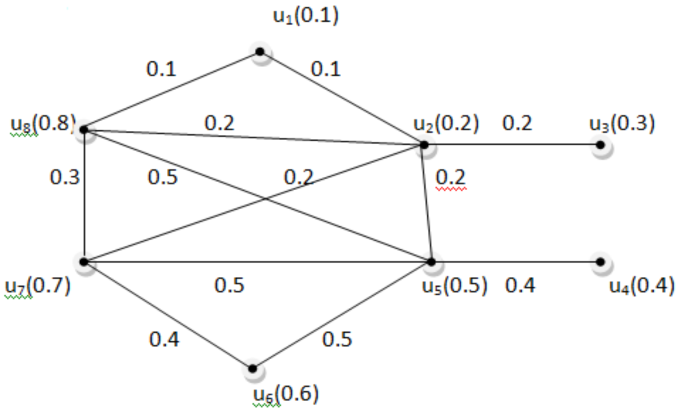
ii) $\gamma^{f}(G) \leq \psi_{ir}^{fcd}(G)$

iii) $\psi_{ir}^{fcd}(G) \geq \max\{\psi_{ir}^{f}(G), \gamma^{f}(G)\}$

Definition: 4.6 domf ND-chromatic

Let G be a fuzzy graph which admits FNDC. Let D be a dominating set of V(G). D is called dom^{f}ND-chromatic if <D> admits FNDC and $\psi_{FNDC}^{f}(<D>) = \psi_{FNDC}^{f}(G)$. Since V is a dom^f ND_chromatic set of G, the existence is guaranteed.

Example: 4.7



Let $\pi^{f} = \{\{u_2, u_5\}, \{u_3, u_4, u_6, u_8\}, \{u_1, u_7\}\}$ is a minimum FNDC partition of G . Let $D = \{u_1, u_2, u_5, u_6, u_7, u_8\}$ is a minimal dominating set . Let $\pi_1^{f}(<D>) = \{\{u_1, u_7\}, \{u_2\}, \{u_5\}, \{u_6, u_8\}\}$ is a minimum FNDC partition of <D>. Thus $\psi^{FNDC}(<D>) = \psi^{FNDC}(G) = 4$, D is a dom^f ND-chromatic set of G.

Definition: 4.8 dom^f ND-chromatic number and upper dom^f ND-chromatic number

The minimum cardinality of a minimal dom^f ND-chromatic set of G is called dom^f ND-chromatic number of G and is denoted by $\gamma^{FNDC}(G)$. The maximum cardinality of a minimal dom^f ND-chromatic set of G is called upper dom^f ND-chromatic number of G and is denoted by $\Gamma^{FNDC}(G)$.

Let us consider the above example , let $D_1 = \{u_1, u_2, u_5, u_6, u_7, u_8\}$ is a dominating set of G and $\psi^{FNDC}(<D_1>) = 3 = \psi^{FNDC}(G)$. D_1 is a minimal dom^f ND-chromatic set of G and $|D_1| = 6$. Thus $\gamma^{FNDC}(G) = 6$

Let $D_2 = \{u_1, u_2, u_3, u_5, u_6, u_7, u_8\}$ is a dominating set with $\pi_1^{f}(<D_1>) = \{\{u_1, u_3, u_7\}, \{u_2, u_6\}, \{u_5\}, \{u_8\}\}$ and $\psi^{FNDC}(<D_2>) = 4 \,\&\, |D_2| = 7$. Thus $\Gamma^{FNDC}(G)$ =7

Remark: 4.9

$$\gamma^{FNDC}(K_n) = n$$

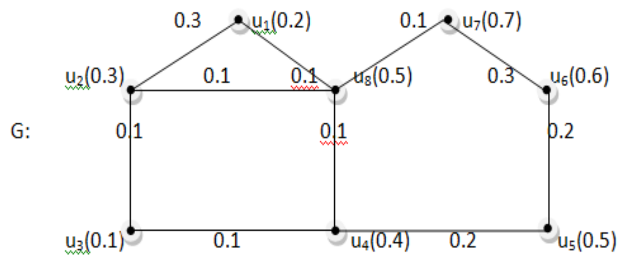
Definition: 4.10 FNDC-critical fuzzy graph

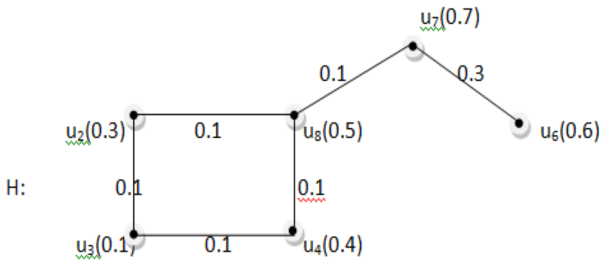
Let G be a fuzzy graph admit FNDC. Then G is said to be FNDC-critical if for every proper fuzzy subgraph H of G is FNDC and $\psi^{FNDC}(H) \neq \psi^{FNDC}(G)$

Remark: 4.11

i) The complete fuzzy graph $K_n, n \geq 1$ is FNDC-critical

ii)





ψ^{FNDC}-partition of G is $\pi^f = \{\{u_1,u_3,u_5,u_7\},\{u_2,u_4,u_6\},\{u_8\}\}$

ψ^{FNDC}-partition of H is $\pi_1^f = \{\{u_3,u_6,u_8\},\{u_2,u_4,u_7\}\}$

Thus $\psi^{FNDC}(G) = 3 \,\&\, \psi^{FNDC}(H) = 2$.Hence $\psi^{FNDC}(G) \neq \psi^{FNDC}(H)$

Therefore G is FNDC-critical fuzzy graph.

Theorem: 4.12

Let G be a fuzzy graph without fuzzy isolates

1. $\gamma^{FNDC}(G) = 1$ if G=K_1
2. $\gamma^{FNDC}(G) = n$ if G is FNDC-critical fuzzy graph

Proof

1. Suppose $\gamma^{FNDC}(G) = 1$. Let D is the minimum dom^{f}ND-Chromatic set of G. Then |D|=1.Thus $\psi^{FNDC}(<D>) = \psi^{FNDC}(G) = 1$. Since G has at most one isolate, then $G = K_1$. Conversely , suppose $G = K_1$. Then V(G) is a dominating set of G with $\psi^{FNDC}(<V>) = \psi^{FNDC}(G)$ and $\gamma^{FNDC}(G) = 1$.
2. Suppose $\gamma^{FNDC}(G) = n$. Let us prove the result by contradiction. Suppose G is not a FNDC critical fuzzy graph . Then there exist a vertex $v \in V(G)$ such that $\psi^{FNDC}(<G>) = \psi^{FNDC}(G - v)$. Let the dominating set $D = V_1 - \{v\}$. Since v is not an isolate , v is dominated by D. Thus , D is dominating set of G and $\psi^{FNDC}(<D>) = \psi^{FNDC}(G - v) = \psi^{FNDC}(G)$. Therefore , D is a dom^f ND-chromatic set of G. Thus, $\gamma^{FNDC}(G) \leq |D| = n - 1$, which is a contradiction to our assumption.

Hence , G is a FNDC-critical fuzzy graph.

Conversely, assume that G is FNDC- critical . Then $\psi^{FNDC}(<D>) \neq \psi^{FNDC}(G)$. Hence $\gamma^{FNDC}(G) = n$.

Definition: 4.13 Chromatic polynomial for FNDC

Let G be FNDC. Let $k \geq \psi^{FNDC}(G)$. The number of ways in which vertices of G can be colored with fuzzy k-colors in such a way that adjacent vertices receive distinct colors with $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ and the codes of the vertices are distinct for distinct vertices is called FND-coloring polynomial for G and is denoted by $\pi_{FNDC}^k(G)$.

Example: 4.14

$$\pi_{FNDC}^k(K_n) = k(k-1)....(k-(n-1))$$

Theorem: 4.15

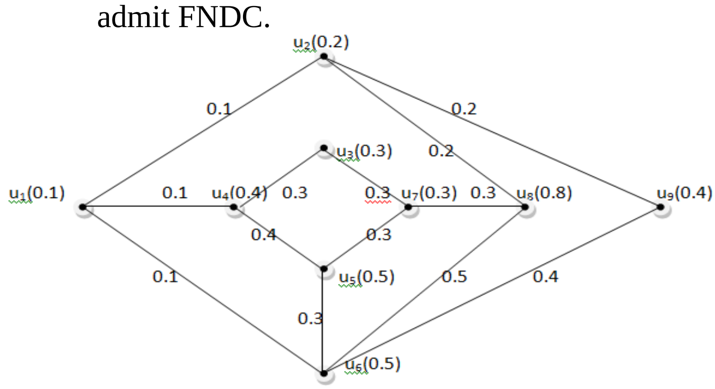
Let G be a fuzzy graph without isolates. Then $\pi_{FNDC}^k(G \cup K_1) = \pi_{FNDC}^k(G).k$

Theorem: 4.16

Let G be a fuzzy graph . Then $\pi_{FNDC}^k(G + K_1) = k.\pi_{FNDC}^k(G)$

Remark: 4.17

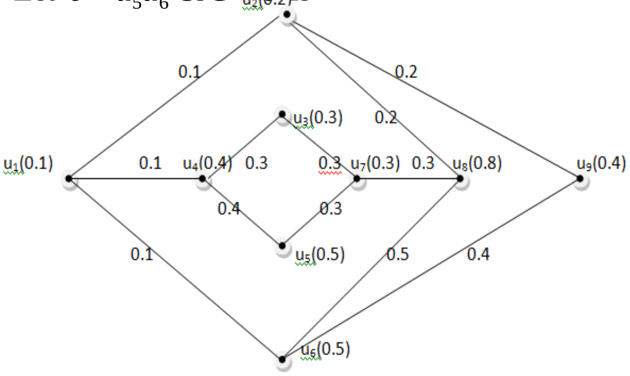
1. Let G be a cycle on n(≥ 5)vertices. Then $G + K_1$ is a wheel on (n+1) vertices and also G and $G + K_1$ is FNDC. Thus $\pi_{FNDC}^k(C_n) = \frac{1}{k}\pi_{FNDC}^{k-1}(W_{n+1})$
2. Let G be a fuzzy graph which admits FNDC. Let e be an edge of G. then G-e need not admit FNDC.



ψ^{FNDC}-partition of G is $\pi^f = \{\{u_1,u_7,u_9\},\{u_2,u_3,u_5,u_6\},\{u_4,u_8\}\}$

G admits FNDC and ψ^{FNDC}(G)=3

Let $e = u_5u_6$ & $G - e$ is

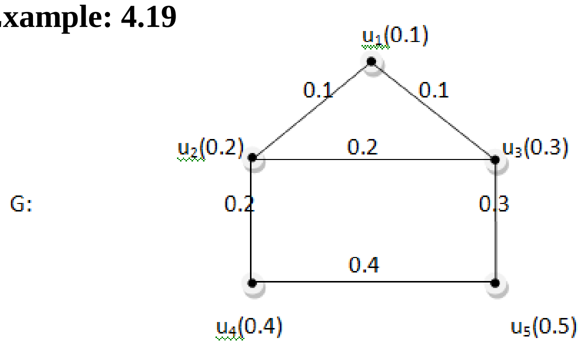


Here u_3, u_5 & u_2, u_6 have same neighborhood set and u_5, u_6 are non-adjacent . Hence G-e is not FNDC.

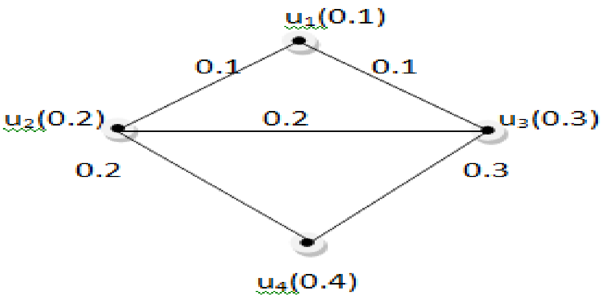
Remark: 4.18

Let G be a graph which admits FNDC. Let e be an edge of G with $\mu(uv) \le \sigma(u) \wedge \sigma(v)$. Then G.e need not be FNDC

Example: 4.19



ψ^{FNDC} - partition of G is $\pi^f = \{\{u_1, u_5\}, \{u_3, u_4\}, \{u_2\}\}$. Let $e = u_3u_5$ and G.e is



Definition: 4.20Chromatic polynomial for irregular fuzzy coloring

Let g be a fuzzy graph and K be a positive integer such that $k \ge \psi_{ir}^f(G)$. Two fuzzy ir-coloring $\{V_1, V_2, ..., V_K\}$ & $\{V_1', V_2', ..., V_k'\}$ are said to be equal if $V_i = V_i'$ such that $\sigma(V_i) = \sigma(V_i')$ for all $1 \le i \le k$ & $\pi_k^{irf}(G)$ denote the number of distinct k fuzzy ir colorings of G.

Corollary: 4.21

$$\pi_k^{irf}(K_n) = k(k-1).....(k-(n-1))$$

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