AN APPROACH TO DETERMINE MAGNITUDE AND DIRECTION ERROR IN GPS SYSTEM.

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ABSTRACT

The Global Positioning System (GPS) allows locating an object in any part of the World with a certain degree of accuracy. Some precision farming activities need to operate with a sub-metric level of accuracy (Grewal et al. 2007). In this article, an approach is introduced to determine, by means of relative positioning, the magnitude and direction of error that the GPS presents. With this error vector it is possible to correct any low cost standard GPS receiver to improve the positional accuracy and to obtain thus more exact distances.

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1. INTRODUCTION

The functioning principle of the GPS is based on measuring ranges of distances between the receiver and the satellites (Xu 2007, Misra & Enge 2010). The GPS has an architecture that is divided into three segments: spatial, control and user’s (Clarke 1998). The first is composed of 24 satellites over 20 thousand kms away from the Earth, divided into 6 orbital levels and each satellite period is of about 12 hours. The second segment is composed of Earth stations distributed around the planet, controlling that satellites do not deviate from their trajectory. Finally, GPS receivers are in the user’s segment, which in general use two frequencies: L1 at 1575.42 Mhz and L2 at 1227.60 Mhz (Feldmann et al. 2009).

There exist mono and multi-frequencies. The accuracy in the horizontal level is of 10-15 meters 95% of the readings (Zandbergen & Arnold 2011). Sometimes, it is more precise, but it depends on a variety of factors that include from the deviation or the delay of the signal provoked by the atmosphere, the bouncing of the signal in buildings or its concealment due to the presence of trees, low accuracy of clocks and noise in the receiver. At the vertical level, accuracy is reduced to 50% regarding the horizontal level (Featherstone 2004). Systems that enhance positional accuracy are: the DGPS (Differential GPS), AGPS (Assisted GPS), RTK (Real-Time Kinematic), e-Dif (extended Differential), amongst others.

a. In the first system, it must be paid an amount for the service and for the DGPS corrections be valid, receiver must be relatively near a DGPS station; generally less than 1,000 km. The achieved accuracy can be of a few meters (Featherstone 2004, Satheesh 2005, Ilčev 2005, and Ghavami et al. 2007). The correction signal can not be received if it is a mountainous zone.

b. In the second one, on the other hand, it is necessary to have mobile devices with active data connection or cell phone like GPRS, Ethernet or WiFi (Ho 2011). It is used in the cases where there is a weak signal due to a surrounding of buildings or trees; this implies having a not much precise position. Standard GPS receivers, in order to triangulate and position, need a certain time of cold start (Li & Wu 2009, Van Diggelen 2009).

c. In the third case, (RTK), it is paid for the service and, besides, it is very expensive to acquire the infrastructure. This is a technique used in topography, marine navigation and in agricultural automatic guidance in the use of measurements of signals carrying navigators with GPS, GLONASS and/or Galileo’s signals, where only one reference station provides correction in real time, obtaining a sub-metric accuracy (Dardari et al. 2012).

d. The last case is autonomous and it needs stations that generate files in RINEX (Receiver INdependent Exchange format) format, a format created to unify data of different receivers manufacturers (Hanif et al. 2012). It generates autonomous corrections regarding a coordinate of arbitrary reference and it extrapolates them in time (GPS World 2002). It is a very consistent relative positioning and its accuracy is of about 1 meter. The system's objective is to study waste from the initializing process to isolate the most important systematic errors that introduce the corresponding equations to each satellite. It is applicable for a reduced time of 40 minutes approximately; since later the
systematic error changes, in this case a new error must be calculated again. The product is mainly aimed at customers in South American, African and Australian regions; where differential corrections are available only by means of a subscription payment.

e. Besides, there exist raise systems that increase accuracy to sub-metric levels. Those based on satellites (SBAS), based on ground (GBAS) and based on aircraft (ABAS) (Grewal et al. 2007). Most of these implementations are used in different applications and some of them are available for users without special permissions. Even then, costs are high due to the need of certain devices with special characteristics or some infrastructure in agreement with the accuracy level aimed at.

Errors produced by the GPS system affect in the same way the receivers located near each other in a limited radius (Grewal et al. 2007). This implies that errors are strongly correlated among near receivers. Thus, if the error produced in one receiver is known, it can be spread towards the rest in order to make them correct their position. This principle is only applicable to receivers that are exactly the same; since, if different, their specifications change so the signal processed by one individual is not the same to that processed by another one. All GPS differential methods use the same concept (Di Lecce et al. 2008). DGPS requires a base station with a GPS receiver in a precise known position. The base compares its known position with that calculated by the satellite signal. The estimated difference in the base is applicable then to the mobile GPS receiver as a differential correction with the premise that any two receivers relatively near experiment similar errors (Zandbergen & Arnold 2011).

In this article it is carried out a detailed analysis of the way to calculate error direction and magnitude of a standard GPS in order to build a DGPS system. In Section 2 techniques to calculate errors are analyzed. In Section 3 algorithms used are described. In Section 4, the last one, conclusions and future work are presented.

2. Techniques for error analysis

The experiment carried out is based on the principle of the adopted methodology by the DGPS but with a low cost standard GPS receiver. In order to get measurements, three Garmin 18X USB GPS receivers are used connected to two notebooks. The base station is composed of a notebook and two of the three GPS receivers; the mobile for the notebook and the remaining GPS receiver. The link between the base station and the mobile one is a wireless connection from point to point link.

In this context, in the base system measurements from the GPS receivers are obtained and after a certain period of time, which is necessary for the system stabilization, two positions are estimated. The positions’ estimation is carried out with a Kalman filter; since an estimation problem with so many noisy redundant data is a natural application for the Kalman filter; this allows using some of the redundant information to remove the effects from the error sources. The Kalman filter is used to eliminate the white Gaussian noise (Eom & Lee 2010). Receivers are placed at a known distance between themselves (relative positioning). At the end of this stage, a cloud of points is obtained with the positions delivered by the receivers. In Figure 1 it is observed the cloud of points from both receivers as well as each of the estimated points E1 and E2 which correspond to the GPS 1 and 2 receivers respectively. The estimated point from GPS 1 is selected as anchor point of the whole experiment. From this, all necessary calculations are carried out with the objective of finding the GPS system error.

Fig. 1. Estimated points E1 and E2.

With both estimated positions (E1 and E2), it proceeds to calculate the distance between them. If the estimated distance is different from the actual one (more/less a threshold) it is detected that there is a positioning error. In the case of Figure 2, the estimated distance with the estimated points is greater than the actual one.

Fig. 2. Estimated distance vs. actual distance.

Besides, Figure 2 shows a circumference with a radius equal to the actual distance measured with a tape measure from GPS 1 to GPS 2 with center in the estimated point for the GPS 1. A circumference is chosen, as GPS 2 can be at that distance but in any point of the circumference. This is the working principle that the GPS system uses to get the receiver’s position (Grewal et al. 2007). After calculating the distance of the estimated points and contrasting it to the actual one, a positioning error is deduced. Once it is known that there is an error, its compute its magnitude and direction. On the one hand, the two estimated points are learnt with which the $E1E2$ straight line is drawn and which bonds them. Equation 1 belongs to the straight line that crosses these two points

$$
\frac{(y - y_1)}{(y_2 - y_1)} = \frac{(x - x_1)}{(x_2 - x_1)} \tag{1}
$$

where the x represents the component of Longitude and the y that of Latitude.
On the other hand, it is known that the GPS 2 is in some point of the circumference with center in the GPS 1 and of radius the distance that is defined at the moment of positioning the two receivers. Equation 2 belongs to the circumference with center in \((x_1,y_1)\) of radius \(r\).

\[
(x - x_1)^2 + (y - y_1)^2 = r^2 \tag{2}
\]

As it was mentioned before, the GPS 2 receiver must be in some point of the circumference with center in the estimated point (E1) for GPS 1 and with the same distance with which GPS 2 receiver was placed. In the case presented, it is observed in Figure 2 that the estimated point (E2) for GPS 2 is farer than the actual distance at which the receiver is placed on ground. Knowing about the equations that define the straight line crossing both estimated points and the radius circumference equation equal to the GPS 1-GPS 2 distance with center in GPS 1, it is proceeded to approximate, by means of the intersection of the straight line and the circumference. This intersection is presented as a polynomial of second degree. It is mathematically solved and \(a\), \(b\) and \(c\) coefficients are obtained (equations 4, 5 and 6). Equation 3 only presents an auxiliary estimate in order not to repeat it in the other operations and to make the rest of the equations more legible. With these coefficients cleared by means of Bascara (equation 7) the roots are found (two because of being of second degree) from the polynomial. From the two roots found, one is chosen and the intersection points are calculated.

\[
divisor = y_2^2 - 2y_2y_1 + y_1^2 \tag{3}
\]

\[
a = 1 + \frac{x_2^2 - 2x_2x_1 + x_1^2}{divisor} \tag{4}
\]

\[
b = -2y_1 + \frac{-2x_1y_1 + 4x_1x_2y_2 - 2x_2^2y_1}{divisor} \tag{5}
\]

\[
c = y_1^2 - r^2 + \frac{x_2^2y_1^2 - 2x_2x_1y_2^2 + x_1^2y_2^2}{divisor} \tag{6}
\]

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{7}
\]

It is worth mentioning that in order to minimize the estimates and be able to normalize, everything is transferred to a same unit (meters) and it is displaced to \((0,0)\) in order to later obtain the difference between E2 and E1.

After having obtained the two roots, only one is taken into account. The nearest to the estimated point of GPS 2 in some of its components is chosen, in this case in Latitude, since the other one is meaningless due to being too far away (on the other side of the circumference).

\[\text{result}_{\text{latitude}} = \text{rootCloser}(y_2, \text{roots.root1}, \text{roots.root2})\]

Finally, the other component (Longitude) is obtained depending on the Latitude found in the previous point as seen in equation 8.

\[
\text{result}_{\text{longitude}} = x_1 + \left(x_2 - x_1\right) \left(\frac{\text{result}_{\text{latitude}} - y_1}{y_2 - y_1}\right) \tag{8}
\]

The corrected point is already there, as shown in Figure 3, being this exactly in the intersection of the straight line and the circumference, as it has been planned.

**Fig. 3. Corrected point in the intersection of the straight line with the circumference.**

With these operations the error magnitude is known but it is also necessary to get the direction of the found error. Error direction can be given in any of the four senses and their combination. Figure 4 shows how the direction is obtained for each case. Four base cases are detected from which other four are derived:

- When the corrected point has greater Latitude than the estimated point and greater Longitude than that of the estimated.
- When the corrected point has greater Latitude than the estimated point and lesser Longitude than that of the estimated.
- When the corrected point has lesser Latitude than the estimated point and greater Longitude than that of the estimated.
- When the corrected point has lesser Latitude than the estimated point and lesser Longitude than that of the estimated.
- When the corrected point has greater Latitude than the estimated point and the Longitude is the same.
- When the corrected point has lesser Latitude than the estimated point and the Longitude is the same.
- When the corrected point has the same Latitude than the estimated point but Longitude is greater.
- When the corrected point has the same Latitude than the estimated point but Longitude is lesser.

**Fig. 4. Determination of found error direction.**
In this analysis negative latitudes and longitudes are found; since the region where the experiment is carried out is to the west of Greenwich meridian and to the South of Ecuador. Because of this, all data and results found are dealt with in the third quadrant as in Figure 4. Therefore, in order to get the error direction it is proceeded to verify which side of the corrected point the estimated point is. If the straight line $\overline{E1E2}$ is horizontal or vertical, some of the components are null, in the horizontal case, Latitude is eliminated and in the vertical the Longitude; that is, there are no corrections in these components since they are not required.

Finally, in Figure 5 it is observed the magnitude and found error direction for the study case introduced.

**Fig. 5. Magnitude and direction error.**

3. Algorithms and used techniques

For the analysis of data, combinations of different techniques and algorithms are used in order to find a better result. In a first processing stage, applied mathematics covers:

- **Static Kalman:** is a set of mathematic equations that provide an efficient recursive solution of the method of minimum squares. This solution allows calculating a linear, unbiased and optimum estimator of the state of a process in each moment of time ($t$) with base on the available information at the moment $t-1$, and update, with the available information at the moment $t$, the estimator value.

- **Dynamic Kalman:** is that system in which the $x$ variable value to be estimated has a value that changes throughout the time ($t$), but these states have some known relationship with the instant $i$ and $i+1$. For example, if an object position is measured, it can be predicted that the position will be:

$$x_{i+1} = x_i + \Delta t \times v_i$$

where $\Delta t$ is the passed time and $v_i$ the speed at instant $i$. Position can be obtained by a GPS, for instance, and speed with an additional measurement element such as the accelerometer.

- **Kalman with adjustment of error standard deviation ($\sigma_R$):** the deviation is modified and checked in order to see which adjust better. This measure is calculated as the square root of variance, which is at the same time the sum of the squares of each error (Table 1) as shown in equation 9. It is worth mentioning that from Table 1 the only error that is not taken into account is that of signal P(Y) arrival; since work is carried out without the precision code.

$$\sigma_R = \sqrt{3^2 + 2^2 + 1^2 + 0.5^2} = 6.7 m$$

(9)

**Table 1. GPS error source.**

<table>
<thead>
<tr>
<th>Source</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arrival of signal C/A</td>
<td>$\pm 3$ m.</td>
</tr>
<tr>
<td>Arrival of signal P(Y)</td>
<td>$\pm 0.3$ m.</td>
</tr>
<tr>
<td>Ionosphere</td>
<td>$\pm 5$ m.</td>
</tr>
<tr>
<td>Ephemeris</td>
<td>$\pm 2.5$ m.</td>
</tr>
<tr>
<td>Satellite clock error</td>
<td>$\pm 2$ m.</td>
</tr>
<tr>
<td>Multipath</td>
<td>$\pm 1$ m.</td>
</tr>
<tr>
<td>Troposphere</td>
<td>$\pm 0.5$ m.</td>
</tr>
<tr>
<td>Numerical errors</td>
<td>$\pm 1$ m.</td>
</tr>
</tbody>
</table>

Now, error standard deviation ($\sigma_v$) in the receiver’s position is estimated, but having into account additionally the PDOP and the numerical error; therefore, the PDOP is added to the calculated deviation from typical errors, since for each measurement taken, this varies according to the instant geometry of satellites. The result of standard deviation used for the Kalman filter is equation 10.

$$\sigma_v = \sqrt{PDOP^2 \times \sigma_R^2 + \sigma_{num}^2} = \sqrt{PDOP^2 \times 6.7^2 + 1^2}$$

(10)

From this comes the fact of applying Kalman with adjustment of standard deviation, since it fluctuates for each piece of information coming from the receivers in each moment as geometry of satellites varies.

- Points average: one of the media limitations is that it is affected by extreme values; very high values tent to increase it while very low values tend to reduce it; this implies that it may stop being representative of the population. It is analyzed but not implemented in the solution.

- **Fuzzy logic:** it is used to determine the error degree that a position has. Rules that determine the error a position presents are related to analyzing some parameters (PDOP, SNR and difference of tracked satellites). The fuzzy system output weights the Kalman filter gain, providing more weight to more precise positions and the other way round.

- Filters allow discarding measurements with much noise or error that influence over the final result of an estimation of a position. Thus, measurements having many errors do not slant the final estimation towards a position far away from the actual one. The application of these filters can be made as measurements are not very far away in time and it is supposed that the Vehicle in which the mobile receiver is placed does not move at high speed; this implies that values do not change radically. High/low step filters are used in an analogical way to the electronic filter.

Since the tool is thought to operate in different places of heterogeneous characteristics, relations and configurations are used in order to be able to customize the use according to needs. Relations and configurations used are the following:

- **Degree/Meters Relation in Latitude:** given the asymmetry, in different places on Earth, the distance that a Latitude degree measures varies.
• Degree/Meters Relation in Longitude: ditto to Latitude.
• Cold Start Time: a start time is considered so the system can be stabilized. In this time, samples of the device are taken and, only at its end, estimation is carried out. The objective is to reduce or soften systematic and random errors of the GPS system.
• Receivers’ distance threshold: it can be determined, in an accurate way, the whole interval of the distance measurement between the base devices. As positioning is relative and its distance is known, it can be added a ± value, since there exists a possibility that an element of distance measurement be not accurate enough. Besides, it reduces the computational load because of not having to process data if distance is within the allowed threshold.

CONCLUSION AND FUTURE WORKS
The technique developed allows to obtain magnitude and error direction provoked by the GPS system. This correction is used by another receiver to correct its own position and thus increase the positional accuracy with the aim of measuring the most precise distances. The experiments carried out with different sets of data provide positions that are used to measure distances and error fluctuates in ± 1 meter the 95% of measurements and in some cases in ± 0,20 meters. The principle is based in mathematical, geometrical functions and filters. As future work, the aim is to increase the accuracy until reaching a maximum error of positioning of ± 0,10 meters. This increase can include the use of another additional signal. Besides, further measurements will be carried out in order to analyze data and to deduce its behavior. It is intended to extend the use in faster vehicles in order to widen the application field of the DGPS system introduced.

REFERENCES