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Evolutionary Game Equilibrium of Noisy Traders in Stock Market - Based on Prospect Theory Perspective

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Abstract: Noise trading can lead to drastic stock price fluctuations and affect investors' rational decisions, which is a persistent problem in the stock market. To explore the evolutionary process of noise traders in the stock market, this paper introduces prospect theory into the evolutionary game analysis process, constructs a payment function different from the expected utility theory, and analyzes the evolutionary mechanism of noise trading and rational investing in the stock market. Theoretical research and numerical simulation results show that: in the stock market, there are multiple possible states of existence for noise traders and rational investors; the proportion of rational investors affects the utility of both strategies, and the distribution of traders' earnings in the market has a greater impact on the equilibrium point; when there is an evolutionary stabilization strategy, the strategy chosen by the first traders determines the direction of evolution; in immature markets, the use of rational investing strategies, higher earnings tend to be achieved.

Keywords:evolutionary game; prospect theory; simulation; replicator dynamics

INTRODUCTION

The problem of the persistence of noise trading in the stock market has been a popular issue for research in theoretical finance. Noise trading in the stock market refers to trading by traders who are misled by false information or influenced by irrational emotions. It is difficult for traditional financial theories to explain the existence and evolution of noise trading. Under the efficient market hypothesis, rational investors will correct the deviation of stock prices and noise traders will eventually exit the market, but in the real market, noise traders exist for a long time and even dominate under specific circumstances. With the development of behavioral finance, the hypothesis of traditional economics rational man has been replaced by limited rationality, and the existence of noise trading has been explained to some extent, but it is still difficult to solve the problems of the behavior pattern and evolution direction of noise trading in the market. Many scholars try to establish mathematical models to analyze the mechanism of noise trading, among which using evolutionary game theory to establish models is a more reasonable approach. The trading mechanism of the financial market and the tendency of both long and short sides to profit are very much in line with the rules of the classical game; the behavior of traders learning higher returns has a high similarity with the law of biological evolution. The evolutionary game theory combines the advantages of both and makes a finite rationality setting for the behavior pattern of the group, therefore, using the method of the evolutionary game to analyze the noise trading in the stock market has more explanatory power and application value. When using the theory of evolutionary games to construct a model of traders' behavior, it is necessary to construct traders' payment functions rationally. A large amount of literature uses the expected utility theory. Still, this theory has shortcomings in empirical studies: firstly, it makes a strict set of traders' risk preferences, which are variable in practice and usually have a more significant relationship with returns; secondly, some axiomatic assumptions often contradict psychological experiments, for example, expected utility theory cannot explain phenomena such as the Allais paradox, the same ratio effect, and the reflex effect. Prospect theory is an alternative and extension of expected utility theory, which is based on psychological research and provides a more reasonable explanation of individual decisions and preferences and can more accurately describe the utility of traders under risky conditions. By combining evolutionary game theory with prospect theory, a more generalizable model can be built to study noisy trading.

Much of the international literature examines noise trading and prospect theory. Andrei Shleifer et al [1] argue that investor sentiment swings and the behavior of limited arbitrage are the causes of market noise and can greatly affect market prices. After the first generation of prospect theory became the theoretical cornerstone of behavioral finance, Daniel Kahneman et al [2] proposed a cumulative prospect theory and constructed an explicit expression of the prospect theory by empirically studying the curvature characteristics of the value function and the weight function. Ulrich Schmidt et al [3] proposed a third generation of prospect theory, which introduced information from different states into the value function and established a prospect theory model with multiple reference points, which greatly improves the application value and prediction ability of prospect theory. Some scholars have used the method of evolutionary games to study the existence of noisy trading. Yi Liu et al [4] established an evolutionary game model using expected utility theory as the utility function, assuming that rational investors are risk-averse and noise traders are risk-averse, and the results proved that the two types of traders can coexist. Bing-Hui Wu [5] improves the model by assuming risk aversion for noise trader's preferences as well, and the results show that the state of the market has an important effect on the evolutionary stabilization strategy. In this paper, the assumption of traders' risk preferences is attenuated, a more reasonable assumption is proposed based on the connotation of noise trading, the types of traders' risk preferences are compatible using prospect theory, and the evolutionary model of noise trading is constructed through the analytical framework of evolutionary games.

MODEL ASSUMPTIONS

To better conform to the definition of noise traders and their behavioral characteristics, a part of the assumptions are revised in this paper based on the research models of some scholars, and the final basic assumptions are as follows.

1. Participants in the securities market can be divided into two types of traders: rational investors and noise traders.
2. Noise trader is a market participant whose demand for risky assets is influenced by irrational beliefs or emotions, and whose irrationality is due to the interference of noise information, which is mainly reflected in the misjudgment of various market states or beliefs about some wrong information.
3. The rational investor (ration investor), in contrast to the noise trader, can obtain information about the current state of the market and evaluate it, free from the emotional influence of certain beliefs, but the size of this category of investors is limited and also risky.
4. Rational investors and noise traders do not differ significantly in their risk preferences; their differences are mainly in the processing of information and the judgment of returns, and their payments (utility levels) in the market conform to the analytical paradigm of prospect theory; in particular, noise traders are unable to identify and process information about the proportions of the two types of traders and the distribution of returns.
5. Traders learn as they trade, shift types, and choose better strategies based on the principle of utility maximization, where the trader's utility is more in line with the structural form of prospect theory.
6. Traders in the market have a certain level of risk tolerance and choose better trading strategies to enhance their utility in the ongoing trading process, and do not exit the stock market or choose risk-free assets because of temporary losses or gains.

Suppose a trader has two pure strategies for trading in the market: rational investing or noise trading. Let the utility of the rational investor be $U(RI)$ and the proportion in the market is x and the utility of the noise trader be $U(NT)$ and the proportion in the market is $1-x$.

MODEL CONSTRUCTION

In this paper, the market participants choose rational investing or noise trading as the action of the game and construct the game payoff matrix for the group of traders, as **Table 1** shown.

Table 1: Game payment matrix

		Participant 2: Rational Investing	Participant 2: Noise trading
Participant 1	Rational Investing	$U(RI),U(RI)$	$U(RI),U(NT)$
	Noise trading	$U(NT),U(RI)$	$U(NT),U(NT)$

The following analysis is done according to the theory of evolutionary games.
The expected return for a participant taking a rational investing is

$$U_{RI} = x\cdot U(RI)+(1-x)\cdot U(RI)=U(RI) \tag{1}$$

The expected return for a participant taking a noise trade is

$$U_{NT} = x\cdot U(NT)+(1-x)\cdot U(NT)=U(NT) \tag{2}$$

The average adaptation for the whole population is

$$\overline{U} = x\cdot U_{RI}+(1-x)\cdot U_{NT} = x\cdot U(RI)+(1-x)\cdot U(NT) \tag{3}$$

The replicator dynamic equation is

$$F(x)=\frac{dx}{dt}=x\cdot(U_{RI}-\overline{U})=x\cdot(1-x)\cdot\left[U(RI)-U(NT)\right] \tag{4}$$

This replicator dynamics equation describes the dynamics of the evolution, in this differential dynamical system, there exist at least 2 equilibria, $x_1=0$, $x_2=1$, and there may be other equilibria $x^*(0<x^*<1)$, x^* which may not be unique and when they exist, satisfy the condition that

$$U(RI)|_{x=x^*}=U(NT)|_{x=x^*} \tag{5}$$

According to the stability theorem of the differential equation, $\tilde{x}$ is the evolutionarily stable strategy when $F'(\tilde{x})<0$

$$F'(x)=(1-2x)\left[U(RI)-U(NT)\right]+x(1-x)\frac{\partial\left[U(RI)-U(NT)\right]}{\partial x} \tag{6}$$

Substituting each equilibrium point, we obtain.

$$F'(x_1)=F'(0)=U(RI)-U(NT) \tag{7}$$

$$F'(x_2)=F'(1)=U(NT)-U(RI) \tag{8}$$

$$F'(x^*)=(1-2x^*)\left[U(RI)|_{x=x^*}-U(NT)|_{x=x^*}\right]+x^*(1-x^*)\frac{\partial\left[U(RI)-U(NT)\right]}{\partial x}|_{x=x^*} \tag{9}$$

By substituting the formula(5) Substitute and simplify to obtain

$$F'(x^*)=x^*(1-x^*)\frac{\partial\left[U(RI)-U(NT)\right]}{\partial x}|_{x=x^*} \tag{10}$$

MODEL ANALYSIS

According to the construction of the above model, there are three possible types of evolutionary stabilization strategies (ESS) for this evolutionary game.

When the external environment changes, there is a constant $U(RI) < U(NT)$, i.e., the prospect value of the noise trader is greater than the prospect value of the rational investor. At this point $x_1 = 0$ is the evolutionarily stable strategy, in this case, over time, all traders in the market will eventually choose the strategy of noise trading, and its evolutionary phase diagram is as follows **Figure 1** The evolutionary phase diagram is as follows

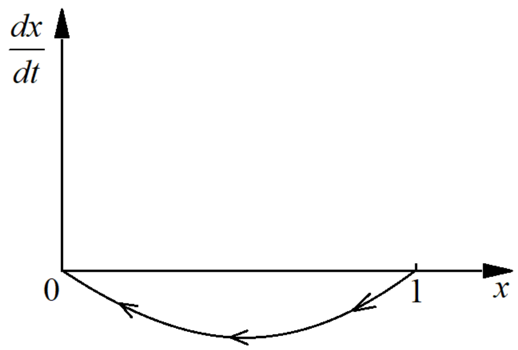

Figure 1: x = 0 for evolutionary stable strategy

Conversely, when $U(RI) > U(NT)$, i.e., the prospect value of the rational investor is greater than the prospect value of the noisy trader. At this point, $x_2 = 1$ is the evolutionarily stable strategy, which means that all traders in the market will eventually choose the rational investing strategy, and the evolutionary phase diagram for this scenario is as follows in **Figure 2**.

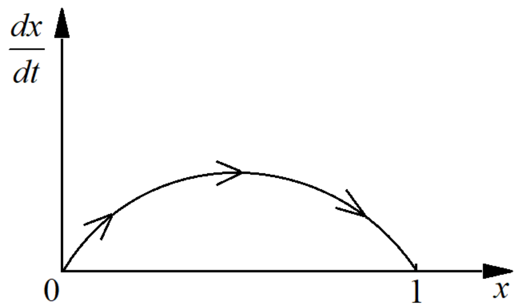

Figure 2: x=1 is the evolutionary stabilization strategy

The ESS is related to the structure of the payment function when there exists x^*, such that $U(RI)|_{x=x^*} = U(NT)|_{x=x^*}$. Since $0 < x^* < 1$, according to Eq.(10), the evolutionary stabilization strategy depends on $sign(\frac{\partial[U(RI)-U(NT)]}{\partial x})$, so that there exists.

When $\frac{\partial[U(RI)-U(NT)]}{\partial x} < 0$, x^* is the evolutionarily stable strategy and is the convergence point, noise traders and rational investors will coexist in the market for a long time, and eventually, the proportion of rational investors in the market will be x^*, and **Figure 3** is the evolutionary phase diagram for only one x^*.

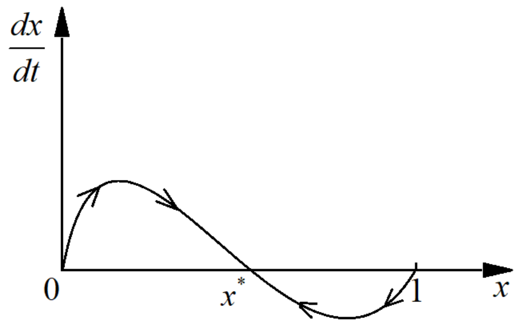

Figure 3: A convergence point for evolutionary equilibrium strategy

When $\frac{\partial[U(RI)-U(NT)]}{\partial x} > 0$, the equilibrium point is the source-out point and the ESS is related to the initial state of the market. If the proportion of the first traders in this stock market choose rational investing is greater than x^*, then all traders will eventually choose rational investing, and when this proportion is less than x^*, noise trading will become an evolutionarily stable strategy, and the evolutionary phase diagram is as follows **Figure 4**.

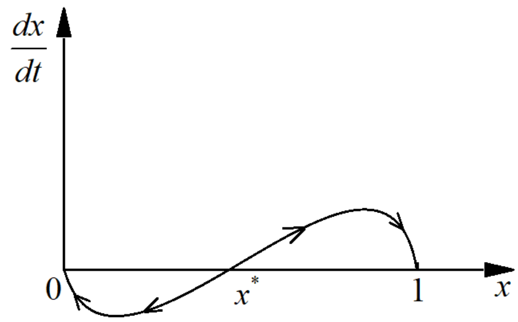

Figure 4: Source out point for evolutionary equilibrium strategy

From the above analysis, it is clear that the existence and stability depend on the construction of the utility. In this paper, the prospect theory is used to construct the payment functions of two types of traders. According to the third-generation prospect theory, the construction of prospect values by multiple reference points can fully reflect the role of information and market states, and the difference between noise traders and rational investors is mainly reflected in the processing and judgment of information. Thus, prospect theory is a more appropriate paradigm for utility construction. Denoting $v(z)$ as the value function, s_i as the state information, $h[s]$ as the reference point function, i.e., the status quo function, $g[s]$ as the behavior function, i.e., the outcome function, and w as the trader's subjective weighting function for events that occur in the state s_i, the prospect values are constructed as follows.

$$U(s_i, g, h) = \sum_i v(g[s_i] - h[s_i])w(s_i; g, h) \tag{11}$$

Under appropriate conditions, simplifying some parameters and functions of this model also works better. When h is a constant behavioral function, one can consider $g(s)-h(s)$ as the benefit and let $z = g(s)-h(s)$, $f(z)$ be the true probability density function of z. Under these assumptions, based on empirical studies of previous generations of prospect theory, the functional form of the value function and the decision weight function can be approximated as

$$v(z)=\begin{cases} z^{\alpha}, z \geq 0 \\ -\lambda |z|^{\alpha}, z < 0 \end{cases} \tag{12}$$

$$w(f)=\frac{f^{\beta}}{\left(f^{\beta}+(1-f)^{\beta}\right)^{\frac{1}{\beta}}} \tag{13}$$

Due to the anomalous trading behavior of noise traders, it can be assumed that: noise traders choose only one anchored state to trade in a phase of trading, as shown by their decision weight of 1, which is not correlated with f; the decision weight function of rational investors is a mapping to the true return distribution. Under these assumptions, the prospect values of the two types of traders are constructed as follows.

$$U(RI)=\sum v(z_i)\,w\big(f(z_i)\big) \tag{14}$$

$$U(NT)=v(z)\,w=v(z) \tag{15}$$

Based on the research results of some scholars, the mechanism of the impact of noisy trading on returns is that the variance of price changes is positively correlated with the trading volume, which is a mixed model consisting of liquidity trading volume and noisy trading volume. Therefore, there is $(1-x)\propto Var(z)$, under which x can be used as a scale parameter of f and the probability density function of returns is $f(z;x)$. According to the above analysis, when x^* exists, there are

$$\begin{aligned} sign\big(F'(x^*)\big) &= sign\left(\frac{\partial\left[U(RI)-U(NT)\right]}{\partial x}\Big|_{x=x^*}\right) \\ &= sign\left(\sum v(z_i)\frac{\partial w}{\partial f}\cdot\frac{\partial f}{\partial x}\Big|_{z=z_i,x=x^*}\right) \end{aligned} \tag{16}$$

The value function and the weight function can be substituted into Eq.(12) and Eq.(13) , some empirical studies in the domestic and foreign literature show that the value function and decision weight function patterns of traders in various markets are broadly similar, and the differences are mainly in the parameters. Therefore, the impact of the distribution of returns on ESS is crucial. In particular, when rational investors trade only once at a stage and the expected return is positive, the direction of evolution is fully determined by the distribution of returns, and when the return decreases as the proportion of rational investors increases, x^* is an evolutionarily stable strategy.

NUMERICAL SIMULATION

Based on the results of the model analysis, this paper uses R software to do simulations of differential dynamical systems under different scenarios.

Scenario 1: When $U(RI)>U(NT)$, i.e., the prospect value of rational investing is always greater than the prospect value of noise trading. Assuming that the difference is 1, the results of 20 simulations are as **Figure 5**,

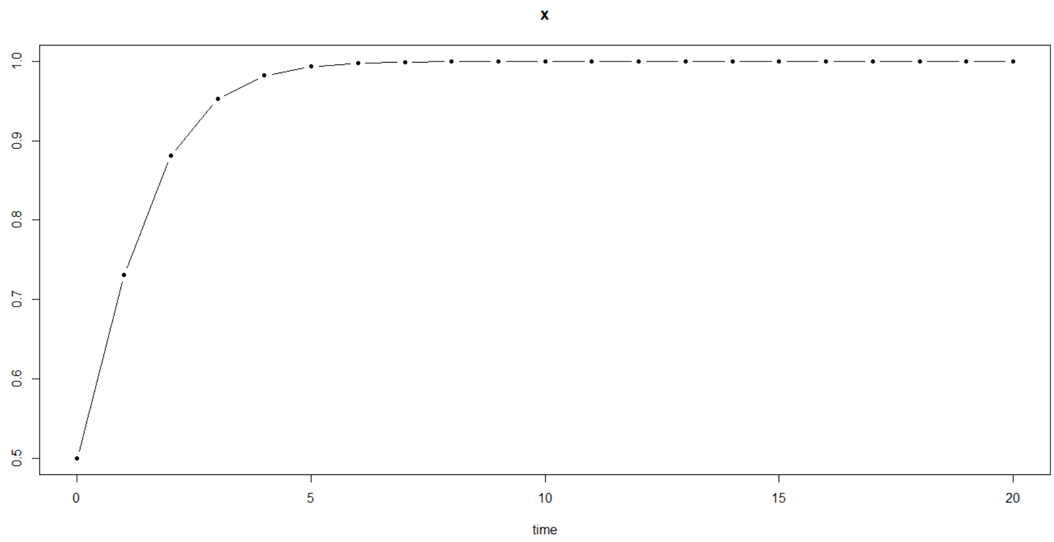

Figure 5: Percentage of rational investors under scenario 1

Scenario 2: When $U(RI) < U(NT)$, the prospect value of rational investing is always smaller than the prospect value of noise trading. Its utility difference is 1. The trend of the proportion of rational investors over time is as **Figure 6**: The trend of the proportion of rational investors over time when their utility difference is 1 is as follows,

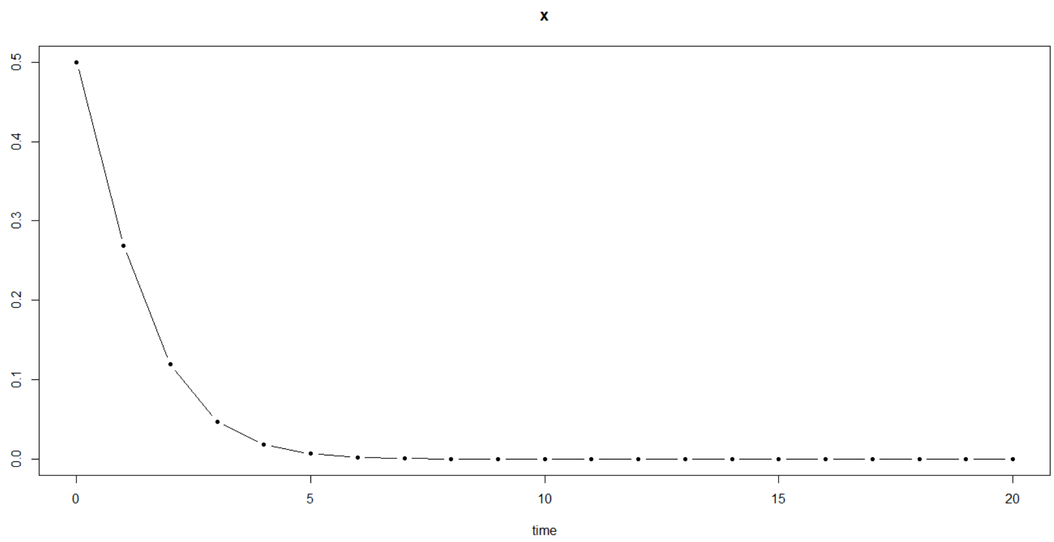

Figure 6: Percentage of rational investors under scenario 2

Scenario 3: When there exists x^* such that $U(RI) = U(NT)$. Assuming that the return obeys a normal distribution with a mean of 0.1, the variance of liquidity trading volume is 0.01, and the variance of noise trading volume is $1\text{-}x$, that is $f \sim N(0.1, 0.01 + (1-x))$, while assuming that the rational investor only decides with a return of 0.5, the probability of obtaining that return is the probability in its small neighborhood, and the prospect value of noise traders is 0.95, according to the summary of risk attitude coefficient, loss avoidance coefficient, and weight coefficient by Li Tiantian[6] , the parameter $\alpha = 0.22, \beta = 0.4$ can be substituted, and the number of simulations is set to 1000. The evolves under different initial conditions as **Figure 7**.

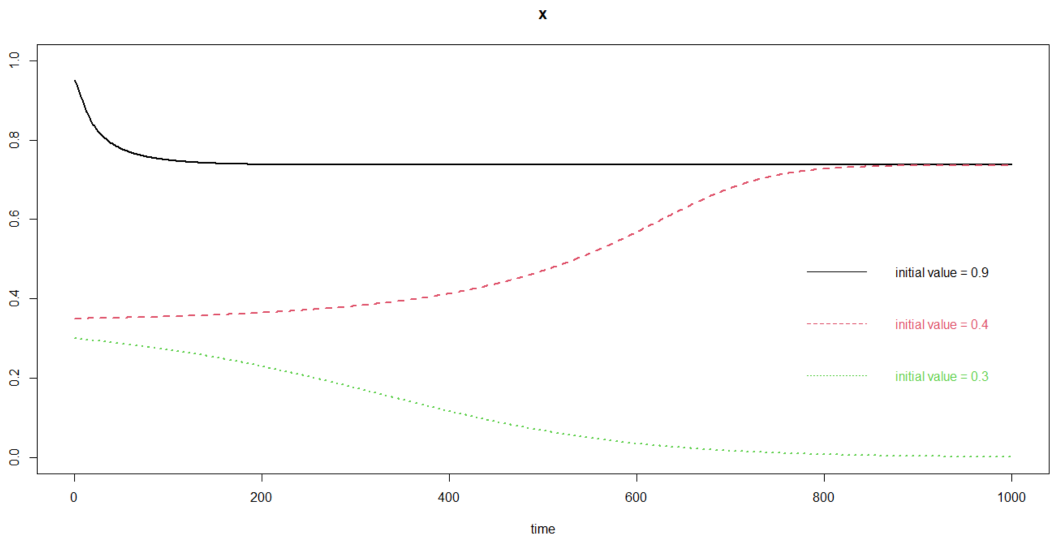

Figure 7: Percentage of rational investors with more balance points

From this figure, we can see that there are two equilibrium points, one is the source-out point of 0.35 and the other is the sink point of 0.7. According to Eq.(16) and the parameters set above, $v(z_i)$ and $\frac{\partial w}{\partial f}$ are both non-negative. The distribution of the returns determines the number and type of evolutionary stabilization strategies, and the initial ratio determines the final result of the evolution.

Figure 8 is a numerical simulation of the proportion of rational investors and their utility at an initial value of $x = 0.5$ and a prospect value of 0.07 for the noise traders. The proportion of rational investors eventually converges at 0.93, and their prospect value increases and then decreases, finally stabilizing at the same prospect value as the noise trader. The return of the noise trader will determine the level of prospect returns of all traders in the final market and will affect the size of the ESS. When the proportion of rational investors does not exceed the ESS, their utility tends to move in the same direction as the proportion.

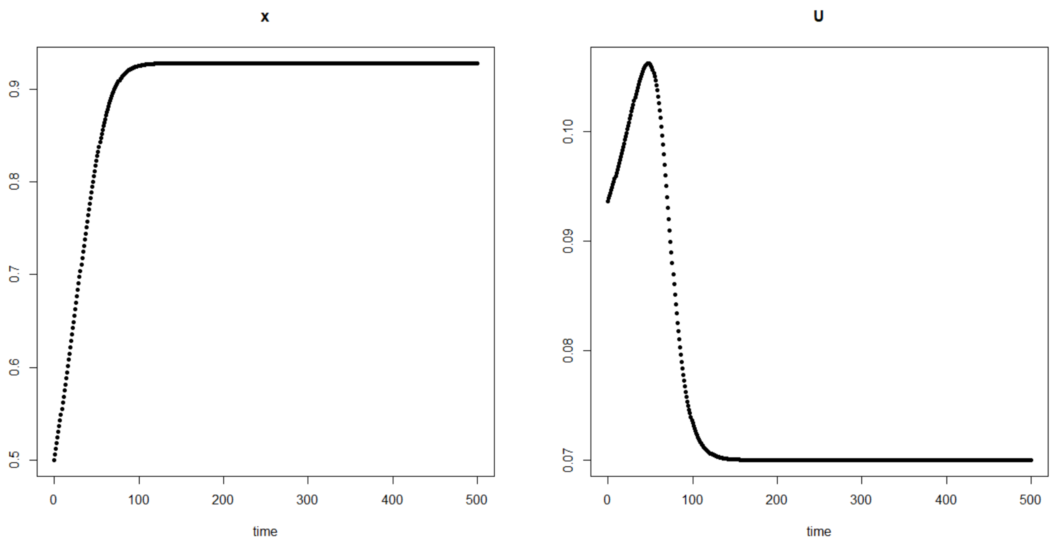

Figure 8: The evolution of the proportion of rational investors and their utility

In each simulation, the proportion of rational investors fluctuates erratically after adding very small, random factors that obey the return distribution to the returns of the two types of traders, and it is difficult to reach a steady state. Assuming that the variance of the rational investor's return is affected only by the risk of liquidity trading volume, the mean value of the prospect of the noise trader is 0.74, close to the mean value of the prospect of the rational investor, and the variance is greater than the variance of the prospect of the rational investor. With such a setup, the process and results of each simulation differ significantly. **Figure 9** is the result of one of the simulations, which indicates that noise traders will exist for a long time and that the rational investor prospect value is positively correlated with its proportion. The higher the proportion of rational investors, the higher their utility, and after reaching a steady-state, their utility volatility increases.

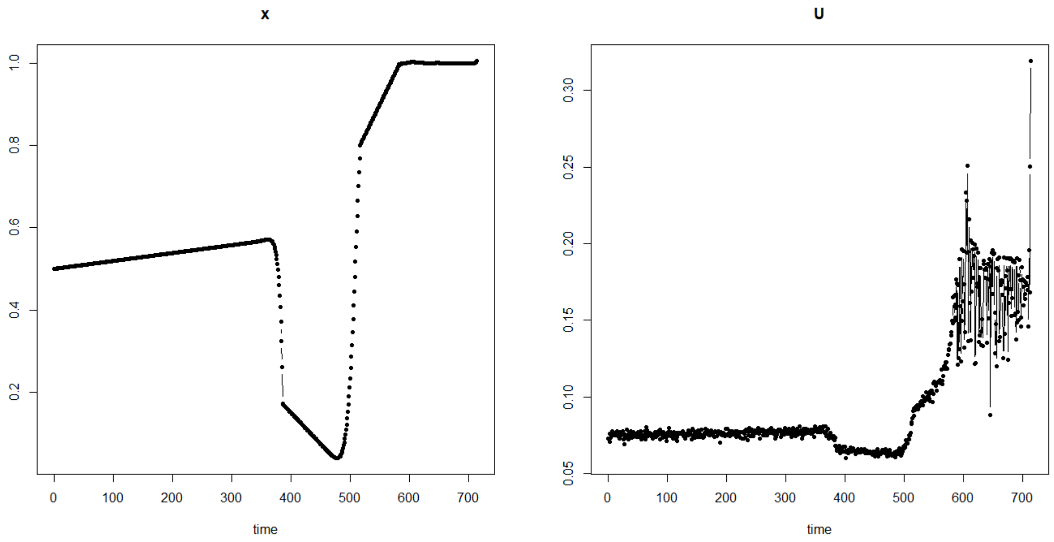

Figure 9: Evolution with random factor interference

CONCLUSIONS AND RECOMMENDATIONS

This paper uses an evolutionary game approach to model noisy trading, and the results show that the use of prospect theory to construct traders' utilities provides a better explanation of the existence and evolutionary equilibrium of noisy trading and is more consistent with actual market conditions. When the difference between the utility of rational investing and the utility of noise trading cannot be reversed, traders will tend to choose the strategy with greater utility; when the utility of both will change with the proportion of rational investors, the distribution of traders' returns will affect the evolutionary stabilization strategy, and eventually, the two types of traders are likely to coexist in the long run. The initial state determines the outcome of evolution, and the utility of noise traders will determine the utility level of the whole market; when there are random factors affecting the returns of both, the direction of evolution of noise traders is more uncertain, and due to the higher uncertainty of the returns of noise traders, when the size of rational investors is controlled within a reasonable interval, they can have a higher utility level than the steady-state, indicating that rational investing strategy can achieve higher returns in immature markets.

Combined with the analysis results, this paper puts forward the following suggestions: first, in order to prevent a part of rational investors from controlling their own size to obtain high returns, the supervision of listed companies on information disclosure should be strengthened to minimize the dissemination of false information in the market, and when listed companies or some research results disclose false information and misleading information, they should be severely punished to enhance the transparency of information for ordinary traders; second, listed companies' initial public When issuing shares, the traders who play new shares should be evaluated, and the proportion of the first group of rational investors should be raised as much as possible, so that the traders in the stock market evolve in the direction of rational decision-making; third, the more rational investors should be guided to invest in relatively mature stock markets, and the regulation of the markets with high price volatility and immaturity should be strengthened to prevent some traders who can obtain and handle inside information from manipulating the trader structure.

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